AE：Hence, I recommend to seriously improve, thereby incorporating the issues raised by the referees (adding some remarks and clarifications is absolutely necessary) and following their valuable suggestions, and re-submit a new, substantially revised version for re-evaluation.

Referee 1：The paper considers the problem of forming the grand coalition for a class of games that involve integer variables, so-called IM games. A common problem is finding a core of a game, an allocation of the total cost among players, such that no group of players chooses to form a sub-coalition. It is known that some games do not have cores and therefore various instruments have been proposed to incentivize the players to stay in the grand coalition. The common instruments are subsidization and penalization.

A more recent instrument proposed by some of the authors of this paper involves simultaneous subsidization and penalization. In this simultaneous framework, the amount of the subsidy provided to the grand coalition is set as a parameter and the corresponding penalty is obtained by solving an appropriate penalty minimization problem, with the idea to incentivize the players to stay in grand coalition but at the same time charge the minimum possible amount of penalty. Exactly this penalty minimization problem is the focus of this paper.

The authors develop an approximate solution for the minimization problem for IM games using Lagrangian relaxation and base the development on another earlier paper, also proposed by one of the authors of this manuscript. Therefore, this paper naturally extends the development of several previously written papers, which adds to other merits of the paper. The methodology is tested using a specific IM game, the TSP game, which has the property of computationally challenging calculation of the characteristic function of the game. This challenge is attempted to be handled by the methodology proposed. Overall, the paper is very well written and makes meaningful contribution to the theory and practice of cooperative games. I will provide the list of comments that hopefully could improve the quality of the paper.

General comments / questions / suggestions:

1. Constraint #2 in problem (4). It appears from the next page that the equality can be replaced to >= inequality in this problem formulation. I would like the authors to expand the discussion of reasons for that being the case **as the brief explanation on page 9 does not seem sufficient**. Moreover, it looks like a good idea to do it right after introduction of problem (4), since in this case it becomes obvious that (4) is a restriction of (2) and therefore z\_r(w) is an upper bound on z(w):

beta(s) <= c\_l(s) + z <= c(s) + z

**beta(V) >= c\_u(U) + z >= c(s) + z**

Hence, the statement of Theorem 1 becomes more obvious and that provides a more illustrative proof of the theorem.

2. When approximate problem (4) is introduced, it is not mentioned which upper bound c\_u(V) is used. Probably a discussion of upper bound possibilities will be good here.

3. Proof of Theorem 1. This is probably a general question but it also directly affects the proof of Theorem 1. The vector of cost allocation beta is defined to be in R^v.

I would like to see some discussion on why some individual cost allocation is allowed to be **negative** (implying a gain for a player, I guess) and how that is possible with existence of grand coalition. My concern is that beta components in the proof of Theorem 1 can become negative, but, again, I am not sure I understand the meaning of negative cost allocation.

4. Proof of Remark 1. I am not able to see "**the point-wise maximum of a finite set of straight lines** (hyperplanes, you mean?)" in defining the z\_r(w) (19). Please clarity.

5. In the description of the Algorithm 1, how is the initial restricted coalition set is constructed? For example, it is not clear what the first step means means if no initial set is defined. **And also, I am not able to see how the initial values of Lagrangian coefficients lambda is constructed and how they change (if they do) during the algorithm**. It would be great to add some clarifications.

6. The description of the TSP game is somewhat different from one can find in the literature. For example in Tamir (1989) the game was defined on an uncomplete graph, while here the game is defined on the complete graph. This affects all the models presented on page 13 and further. Probably both versions of the game exist, but I would like to understand why the descriptions are different.

7. Constraint (15): I am not sure why would one keep (15) in such an aggregated format when it is possible to disaggregate it to

x\_{ij} <= \gamma\_i

x\_{ij} <= \gamma\_j ?

In terms of LP relaxation disaggregation gives a tighter bound and probably will lead to the improvement to Lagrangian relaxation just as well.

8. The symmetry of TSP game. I would like to attract the attention of the authors to the Dantzig Fulkerson Johnson (1954) paper, which originated the development of the TSP theory. Most importantly, the authors there also considered a symmetric TSP problem. If the problem is symmetric, one does not need as many binary variables as was introduced by authors. For example, on page 13 x\_{ij} exists together with x\_{ji} but direction of the travel is not important for symmetric problem **therefore it is sufficient to introduce x\_e for e being an edge or x\_{ij} for i < j only**. This is how the TSP problem was introduced in DFJ and this is something that can simplify many notations in this paper. Also, it will be probably a good idea to get rid of x\_{ii} variables on page 13 and other optimization problems.

9. I am a bit confused by Figure 2. Does it represent 4 different games? What exactly is on y axis and x axis, Penalty and Subsidy on y and x axis for all 4 figures? I suggest to add definitions of **squared points and round points to the legend of the figure**. Why is it that one figure **gets two points and another gets 6** and they are evaluated at different levels of subsidy?

10. In regards to results of experiments in Table 3, **what exactly was used and the upper bound \phi(N)**? and finally a general remark on the lower bound construction z(w):

-- if we consider problem (4) as a restriction of (2), then it is also possible to create a relaxation of the problem in the similar was as problem (4) was constructed.

Specifically, I would imagine that c\_l and c\_u should switch their places but the authors may check this is more detail. Would it be not possible to develop the lower bound using the same (similar) methodology as for the upper bound?

Minor comments:

1. Problem (2) in the paper is frequently called an **LP problem or a combinatorial optimization problem** (page 4, for example). I suggest to use one terminology approach for consistency. For example, page 8 "solve LP (4) with some conventional combinatorial optimization techniques" sounds confusing.

2. Remark 1: probably, for any s in S \ V in the first line of the Remark?

3. Page 8, line 58. Perhaps it is worth mentioning that this is **due to Lagrangian being concave function**.

4. Page 10, line 60. Perhaps LP (4) instead of (2)?

5. Page 10, line 26. Perhaps, it is better to replace forward reference (9) by "reduced cost".

6. Page 13, I would suggest to make a reference to the Dantzig Fulkerson Johnson (1954) paper with respect to constraints (13), as these constraints are not given in TSP

literature but are in fact result of development by those authors.

7. I would suggest merging first two constraints in (11) as it is a bit confusing in its present form.

8. Page 15, line 45. don't you need to remove the word 'minimum' here?

I hope that these comments and suggestions will help the authors to improve the paper.